

Dielectric-induced counterion partitioning and its effect on membrane rigidityJeremy Schmit,^{*} Rebecca Menes, and Philip A. Pincus[†]*Materials Research Lab, University of California, Santa Barbara, California 93106*

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We report on a nonthermal mechanism for counterion partitioning driven by dielectric contrasts in a charged system. This mechanism allows objects to remain charged even in very low dielectric mediums and at low temperatures, if a nearby high dielectric environment can host its counterions. This nonthermal counterion partitioning leads to an attractive counterion pressure that is usually much larger than the van der Waals forces. In the context of a lipid membrane, this partitioning will stiffen the membrane by $\sim k_B T$.

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I. INTRODUCTION

The growing interest in the effects of electrostatics in biological systems has led to a series of questions regarding the role of counterions, given their ability to disperse in space and affect the resulting interactions. It is becoming evident that many of the physically interesting phenomena in these biological, and biologically inspired, systems result from the ability of the small ions to organize in response to local forces. These include both direct interactions and activities of these ions, but also effective interactions such as the fluctuation-correlation induced attractions between like charged objects in aqueous solutions [1,2].

One of the underlying assumptions in treating these systems is that ion distributions are driven by two competing forces: On one hand, the ions are electrostatically attracted back to the larger molecules from whence they were ionized, while on the other, their entropy drives them away. In this paper, we are concerned with yet another mechanism that drives ions away from the ionized source: A process not due to temperature but rather due to dielectric contrast. Ions might favor a distant but polar region in space if their ionized source is submerged in a low dielectric material.

One of the places where this question arises is when we ask how proteins and other charged objects are incorporated into membranes [3,5]. Because of the large difference in the dielectric constants, we usually assume that proteins ionize in water but not within a membrane. Thus, when embedded in membranes they either leave their charged regions exposed in the water while settling their hydrophobic parts in the membrane [3], or if completely submerged in the low dielectric membrane, we assume that they effectively lose their charge by recombining with their counterions. The reason for this is that in the membrane, the electrostatic self-energy of a free ion is about 40 times higher than it is in water. However, in this argument, we have not taken into account the fact that the membrane is living within an aqueous environment and that the counterions have three real

options: to remain very close to the ionized molecule, to disperse within the membrane, or to disperse further away in the high dielectric water surrounding the membrane. In this paper, we will discuss when the third option is available and how it affects the system.

This mechanism for counterion partitioning is not thermally driven and induces an attractive pressure on the low dielectric film (e.g., membrane, oil layer), which is typically much larger than the pressure due to van der Waals forces. In a previous paper, we have shown how the effects of dielectric contrast also modify fluctuation induced interactions in membranes and in which regimes they can dominate [2].

Here, we introduce a simple model for a membrane that demonstrates how the presence of dielectric contrasts can lead to ion partitioning even at the low temperature limit. We calculate the pressure resulting from the partitioning and compare it to the van der Waals forces. Using this model, we show how counterions can use a hydration shell to shield themselves from the low dielectric and how this affects the ion distribution. We also calculate the contribution this ion partitioning makes to the membrane flexibility.

II. MODEL

Consider a planar layer of dielectric ϵ_o and thickness $2d$ embedded in an infinite medium of dielectric ϵ_w with an ionizable layer fixed at the middle plane of the dielectric slab (Fig. 1). In order to simplify the model, we assume that we are at a low temperature limit where the entropy is irrelevant. In the following, we will show that for most systems with strong dielectric contrast, this assumption is reasonable even at room temperature. In this case, there are two relevant contributions to the electrostatic energy of the system: the interactions between counterions and fixed charge on the middle layer, and the self-energy of the counterions depending on their dielectric environment. At zero temperature, the ions will be at one of the two places: either at the surface of the charged object within the low dielectric oily layer, or in the aqueous environment at an optimal distance, Δ , from the dielectric boundary [4]. Thus, the contribution to the energy per unit area of the system due to the interactions between charges is simply the energy of a capacitor filled with two different dielectrics. In CGS units this takes the form

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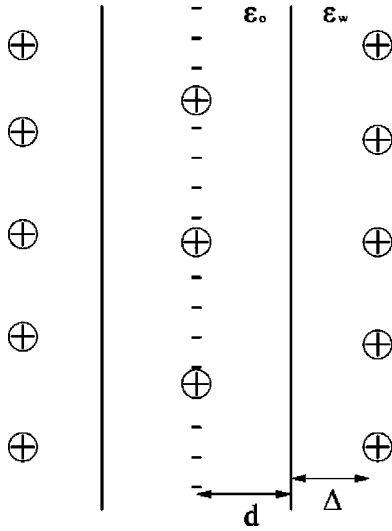


FIG. 1. An ionizable surface coated with an oily layer of thickness d submerged in water. The counterions are partitioned between the close nonpolar region and the more distant polar region.

$$U_{cap} = 2 \left(\frac{2\pi\sigma^2 e^2 d}{\epsilon_o} + \frac{2\pi\sigma^2 e^2 \Delta}{\epsilon_w} \right), \quad (2.1)$$

where σ is the number density of charges in the aqueous medium on either side of the membrane, e is the charge of an electron, and the factor of 2 accounts for the fact that there is a capacitor at each dielectric interface. The self-energy difference between a charge at the center of our dielectric slab and a charge at a distance Δ from the edge of the slab is given by

$$f_{SE} = -\frac{e^2}{2\epsilon_o a} \left(1 - \epsilon - 2\frac{a}{d} [\ln(2) - \epsilon] - \epsilon \frac{ad}{\Delta^2} \right), \quad (2.2)$$

where $\epsilon = \epsilon_o/\epsilon_w$ and we have truncated at the first order in ϵ . Here, a is an effective ion size introduced in order to estimate the change in self-energy of the counterions between the polar and oily environments. The estimate here assumes that the water and oil environments are uniform, continuous, dielectric mediums characterized by the dielectric constants ϵ_w and ϵ_o and that the ion size does not depend on this environment so that we can use the standard expressions. Similar expressions were used [7,8] in order to estimate the energy of an ion crossing a low dielectric membrane. This is a very crude estimate for the self-energy, which depends sensitively on the structure of the immediate surroundings of the ion [9]. However, it is sufficient to obtain an estimate of the order of magnitude of this energy. Note that the self-energy per charge in oil can also be written in the following form: $(1/2)k_B T(\epsilon_w/\epsilon_o)(l_B/a)$, where $l_B = e^2/\epsilon_w k_B T$ is the Bjerrum length at room temperature (~ 7 Å). This expression clearly demonstrates that even at room temperature, the self-energy of an ion in oil is about two orders of magnitude bigger than its entropy which is of the order of $k_B T$, and therefore negligible.

Minimizing the sum of Eqs. (2.1) and (2.2) with respect to σ and Δ yields $\sigma \approx 1/8\pi ad$ and $\Delta \approx (4ad^2)^{1/3}$, to the lowest

order in ϵ and a/d . Note that for layers of thickness $d < d_c \approx 1/4\pi a\sigma_0$, where σ_0 is the concentration of ionizable groups in the membrane, the surface becomes fully ionized and the counterions are all in the unbound state at the oil-water boundary. When the layer becomes thicker than this critical value, the counterions start binding with the charged plane and the effective charge decays. For typical, high, membrane chargings, d_c is of the order of 1 nm.

A. Counterion pressure

The partitioning of counterions outside the oil layer leads to a pressure on the layer. This pressure is calculated by using the result for the unbound charge, σ , in the energy [Eqs. (2.1) and (2.2)],

$$\Pi(d) = \frac{\partial U}{\partial d} = k_B T \frac{1}{8\pi} \frac{\epsilon_w}{\epsilon_o} \frac{l_B}{a^2 d^2} \left[1 + O(\epsilon) + O\left(\frac{a}{d}\right) \right]. \quad (2.3)$$

The pressure tends to squeeze the oil layer and its strength increases as the ratio of dielectric constants increases. The pressure decays (as do the number of unbound counter ions) as the layer thickness, d , increases. The $1/d^2$ decay is weaker than that of the van der Waals forces ($\sim 1/d^3$), and therefore dominates over large length scales. For typical values of the Hamaker constant ($A \sim 25k_B T$), we find that the ratio of the counterion pressure to that of the van der Waals force is of the order of $10-100d/\text{Å}$, so that for any physically realizable distances, the counterion attractive pressure dominates over the van der Waals pressure. This pressure can be directly related to the instability that leads to dewetting of an oil film when water is added to a charged system [6].

B. Shielding by water drops

The model we presented should be modified in order to allow the ions to carry with them a hydration shell of water when residing in the oily layer [7]. This drop will lower the self-energy of the ions in oil and will affect the resulting charge partitioning. Using a rough macroscopic argument, the size of the drop is determined by a balance between the electrostatics, which is a driving force for a large drop and the oil (water) surface tension, γ , which tends to decrease the size. This balance yields

$$R_{w.d.} = \left[\frac{1}{16\pi\epsilon_o} \frac{e^2}{\gamma} \left(1 - \frac{\epsilon_o}{\epsilon_w} \right) \right]^{1/3} \approx 4 \text{ Å}, \quad (2.4)$$

which is consistent with a few water layers.

The effective new self-energy in oil is given by

$$U_{self} \approx \frac{3}{4} \frac{e^2}{\epsilon_o R_{w.d.}}, \quad (2.5)$$

and is about four times smaller than the bare self-energy in oil. Effectively, we can absorb this into the ratio of dielectric constants which goes from $1/40$ to $1/10$. This will slightly modify the above results. In this calculation of the drop size

and the new effective self-energy, we used the same approximation, which assumes that the two mediums can be modeled as uniform dielectrics.

C. Membrane rigidity

Finally, we wish to determine the effect of this ion partitioning on the flexibility of the membrane. The elastic properties of a fluid membrane are characterized by three macroscopic parameters—a bending elastic modulus k , a Gaussian modulus \bar{k} , and a spontaneous curvature c_0 . The deformation free energy per unit area, expressed in terms of the mean curvature H and Gaussian curvature K , may be given by the Helfrich free energy [10,11]:

$$f = 2k(H - c_0)^2 + \bar{k}K. \quad (2.6)$$

Within an additive constant, the free energy per area of a spherical membrane with radius R is given by $f_s = (2k + \bar{k})/R^2 - 4kc_0/R$ and a cylindrical membrane with radius R by $f_c = k/2R^2 - 2kc_0/R$. Therefore, the parameters k , c_0 , and \bar{k} may be determined by expanding f_s and f_c around $1/R$ and matching the coefficients.

In the present case, the capacitance terms for spherical and cylindrical geometries can be written as

$$U_{sph} = \frac{e^2}{8\pi R^2 \epsilon_o} \left[q_i^2 \left(\frac{1}{R-d} - \frac{1}{R} \right) + q_o^2 \left(\frac{1}{R} - \frac{1}{R+d} \right) \right], \quad (2.7)$$

$$U_{cyl} = \frac{e^2}{2\pi R \epsilon_o} \left[q_i^2 \ln \left(\frac{R}{R-d} \right) + q_o^2 \ln \left(\frac{R+d}{R} \right) \right], \quad (2.8)$$

where the i,o subscripts refer to the internal and external sides of the membrane and q is the total number of charges in the aqueous medium on the corresponding side of the membrane.

As expected, we find that $c_0 = 0$ due to the symmetry of the system, and the bending moduli are found to be

$$k = \frac{e^2 d}{96\pi \epsilon_o a^2} \left(1 - 4 \frac{a}{d} \ln(2) - 2\epsilon \right), \quad (2.9)$$

$$\bar{k} = \frac{-e^2 d}{48\pi \epsilon_o a^2} \left(1 - 4 \frac{a}{d} \ln(2) + 2\epsilon \right). \quad (2.10)$$

For d a few nanometers long, a a few Angstroms, and the modified dielectric ratio discussed above, we find that k is of the order of $10k_B T$. When d is below the critical value, all the ions are unbound, so the self-energy term remains constant. In the simplest case, half of the charges lie on each side of the membrane. Expanding the capacitance energy, we find that

$$k = \frac{2d^3 \pi \sigma^2}{3\epsilon_o} (1 + \epsilon C_1), \quad (2.11)$$

$$\bar{k} = -\frac{d^3 \pi \sigma^2}{3\epsilon_o} (1 + \epsilon C_2), \quad (2.12)$$

where

$$C_1 = \frac{2}{(\pi^2 d^4 \sigma^2)^{1/3}} + \frac{1}{2\pi d^2 \sigma} + \frac{5}{2(\pi d^2 \sigma)^{1/3}}, \quad (2.13)$$

$$C_2 = \frac{1}{d^2 \pi \sigma} + \frac{3}{(d^2 \pi \sigma)^{1/3}} + \frac{8}{(d^4 \pi^2 \sigma^2)^{1/3}} - \frac{5}{(d^2 \pi \sigma)^{2/3}}. \quad (2.14)$$

When the charges are allowed to move across the membrane, k is reduced by roughly a factor of 16 and \bar{k} is reduced by a factor of 4.

III. DISCUSSION AND CONCLUSIONS

In conclusion, we have shown that under appropriate conditions, molecules can ionize in a low dielectric medium. This will occur if the ions can disperse in a nearby high dielectric material. The fraction of counterions that separate from the ionized source depends on the ratio of the dielectric constants of the polar and nonpolar materials and on the thickness of the nonpolar layer surrounding the charged object. The thinner the layer, the more favorable it is for the counter ions to disperse in the polar region in order to lower their electrostatic self-energy. The degree of partitioning is also affected by the possibility of counterions staying in the nonpolar region by engulfing themselves in a polar hydration sphere. The size of the drop is determined by the dielectric contrast and by the surface tension between the two materials. Counterion partitioning due to this nonthermal effect leads to an attractive pressure on the nonpolar layer, which in most physically relevant cases is much larger than van der Waals forces. In a membrane, this ion partitioning leads to a stiffening of the membrane comparable to thermal fluctuations. Such partitioning effects are also important in systems that show leaching of ions from glassy surfaces submerged in water.

Although we have demonstrated that nonthermal ion partitioning can occur and have important consequences, our calculations neglected several properties of real world systems that may play a significant role. Future work will need to account for finite temperature, the discrete nature of the stationary charges, fluctuations, and include more accurate descriptions of the dielectric properties of the system.

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